Cover page

Title:

SHM concept applied to dynamical defect identification in electrical circuits

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ABSTRACT

The analogies between mechanical and electrical systems are well recognized and established. Constitutive relations for analogous quantities, as well as equilibrium equations, are of the same mathematical form, hence – the behaviour of both systems, under the equivalent assumptions, is also of the same form and can be analysed by employing similar methods. In the paper the focus is put on the system of analogies between electrical circuits and linear truss structures subjected to dynamical excitation. The so-called Impulse Virtual Distortion Method is used to analyse the elastic wave propagation in the truss structures. IVDM allows modeling the modifications of the structure by the state of the time-dependent distortions imposed on the initial structure. Distortions, identified at the given time interval with the state of initial deformations, have a form of sequence of impulses. IVDM states that the dynamical response of the modified structure can be expressed as a superposition of the health structure response and the response generated by the distortions. In the process of identification the measured response in certain elements of the structure (subjected to some known excitation) is used to determine the distortion functions, which in turn define the influence of modification on the structure response. It is demonstrated that basing on the compatible system of analogies the same identification procedure can be also applied to the electrical circuits.

INTRODUCTION

The analogies between electrical and mechanical systems, primarily visible in the mathematical form of equations describing system state and behaviour, are well recognized and frequently mentioned in the literature, for example in works concerning vibrations [1]. One of the main practical use of this concept was searching for the response of the real mechanical systems subjected to specific, hard-to-realize excitations and how the response is influenced by the modifications of properties or re-configuration of the system. Using analogous electrical systems made this process possible and much simpler.

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Along with the development of numerical methods of analysis (like FEM) this approach became unpractical and obsolete. However, there is still a need for the experimental verification of numerical models and analogous systems, although indirectly, may be the answer for that demand. Another field of application, which is the aim of our research, are electro-mechanical coupled systems. The idea is that electrical mesh, embedded into structural element and topologically compatible with its geometry, could have been used to identify and locate mechanical defects (like cracks, whose occurrence would have induced breaks in electrical connections). This identification system is considered not as a stand-alone, rather as working together with other SHM system, so having and applying the same numerical tools for dynamical analysis of both systems would have been a great facilitation. In the following study we concentrate on the numerical model of an electrical circuit, analogous to the FEM-based model of the plane truss structure. This analogy is simple and particularly apparent in the aspects of topology description (members connected in nodes), constitutive relations (forces/displacements vs. currents/potentials) and equilibrium equations (equilibrium of forces and continuity of deformations vs. Kirchhoff's laws). It allows to analyse the proposed model on the basis of the so-called Impulse Virtual Distortion Method, which in the case of truss structures has shown its efficiency [2,3]. Using of IVDM, as it strongly utilizes the superposition principle, is limited to the linear problems. The concept of using Virtual Distortion Method in static analysis and defect identification in the electrical circuits was presented in [4].

NUMERICAL MODEL OF ELECTRIC CIRCUIT

In the proposed model every branch of the electrical circuit corresponds to the truss member and is represented by a two-node finite element. The general Electrical Finite Element (EFE) shown in fig.1 consists of capacitance C, conductance G and inductance L connected parallel. The constitutive relation for the element is:

$$i(t) = Cu'(t) + Gu(t) + \frac{1}{L}\int u(t)dt \tag{1}$$

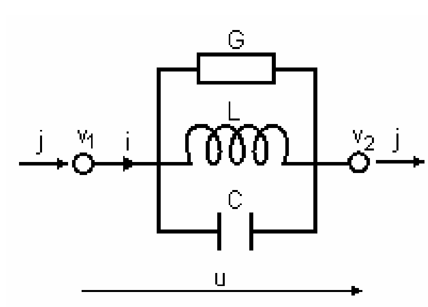

Figure 1. Electrical Finite Element

This parallel model is geometrically compatible with 1DOF mechanical system. Currents associated with G, L and C are equivalent to the forces while voltage corresponds to the velocity. In the FEM model of truss structure the unknown variables are displacements of the nodes, while all excitations are reduced to the nodal forces. The equivalent quantities in electrical model are nodal potentials v (unknown variables) and current sources j supplying the nodes of the circuit (excitations). The model of the circuit is a closed one (no current flow from and on outside), so current sources are associated with the elements. Comparing the EFE with its mechanical counterpart of plane truss element the following differences can be distinguished:

- Every node has only one degree of freedom.
- No geometrical transformation from local to global coordinate system is needed. The directions of current flows and polarization of voltages can be determined with reference to the pre-established oriented graph of the circuit.
- EFE can consists of arbitrary combination of G, L and C (as exist real elements with only one of those features)
- Boundary conditions correspond to the grounding of the nodes (potentials equal zero) and are applied in the same manner as in mechanical models with blocked degrees of freedom. At least one node of the circuit has to be grounded in order to obtain non-singularity of global matrices.

The global equilibrium equation of the system can be obtained by aggregation of the elements (like in FEM), but we will approach to this problem basing on topological information about the structure of the circuit. Circuit structure can be represented in the form of the oriented graph, which in turn can be coded in the form of nodal matrix **M**. Rows of the matrix correspond to nodes while columns to the edges of the graph. $M_{ij} = 1$ means that edge j is adjacent with node i and is oriented from that node, $M_{ij} = -1$ means that edge is oriented toward the node, and zero means that there is no adjacency between edge and node. The starting point in determining the global equilibrium equation is Kirchhoff's current law (algebraic sum of currents entering and exiting node equals zero). In the matrix notation this can be written as:

$$\mathbf{M\,i} = -\,\mathbf{M\,j^e} \qquad (2)$$

On the left-hand side of equation (2) are currents associated with the elements, on the right-hand side – currents associated with the excitations. The system of constitutive relations (eq.1) for all elements looks as follows:

$$\mathbf{i} = \mathbf{C^e u'} + \mathbf{G^e u} + \mathbf{K^e} \int \mathbf{u} \qquad (3)$$

where $\mathbf{C^e}$, $\mathbf{G^e}$ and $\mathbf{K^e}$ ($K^e_{(i,i)} = 1 / L_{(i)}$) are diagonal matrices of parameters. Voltages can be expressed in function of nodal potentials:

$$\mathbf{u} = \mathbf{M^T v}; \qquad \mathbf{u'} = \mathbf{M^T v'}; \qquad \int\mathbf{u} = \mathbf{M^T} \int\mathbf{v} \qquad (4)$$

Substituting 4 into 3 and 2 we obtain:

$$\mathbf{M\,C^e\,M^T\,v'} + \mathbf{M\,G^e\,M^T\,v'} + \mathbf{M\,K^e\,M^T} \int\mathbf{v} = -\mathbf{M\,j^e} \qquad (5)$$

Introducing the following substitution: $\mathbf{v} = \mathbf{x}'$ and marking:

$$\mathbf{C} = \mathbf{M}\,\mathbf{C_e}\,\mathbf{M^T}\;;\quad \mathbf{G} = \mathbf{M}\,\mathbf{G_e}\,\mathbf{M^T}\;;\quad \mathbf{K} = \mathbf{M}\,\mathbf{K_e}\,\mathbf{M^T}\;;\quad \mathbf{j} = -\mathbf{M}\,\mathbf{j_e} \tag{6}$$

we finally obtain the global equilibrium equation, equivalent to the equation of motion in mechanics:

$$\mathbf{C}\mathbf{x}'' + \mathbf{G}\mathbf{x}' + \mathbf{K}\mathbf{x} = \mathbf{j} \tag{7}$$

Because of the assumption that in some elements some features can be neglected matrices **C, G, K** may be singular. Although the direct indication that quantities **i, j, u, v** and **x** are functions of time was discarded, all above relation are fulfilled in every instant. The dynamic response of the system in discrete time domain can be obtained by direct integration of equation (7) according to the Newmark algorithm. Particularly important in the further analysis is finding the response of the system on impulse excitation. This is done by setting the excitation value to 1 in the chosen element in the first time step and to zero in all the following steps, with initial conditions equal zero (non-energized state). It will be shown later that knowing the impulse response it is possible to find the response on arbitrary time-variable excitation.

IVDM FORMULATION

The Impulse Virtual Distortion Method can be classified as a fast re-analysis technique, based on the FEM model of the system. Suppose that the system is subjected to some known excitation, which generates the dynamical response defined as “linear response” of the system. Introducing modifications of the system properties causes that the same excitation generates another, “modified” response. IVDM states that modifications of the system can be modelled by the states of distortions imposed on the initial system. Distortions, defined as the time-dependent states of non-static deformations, in FEM model are introduced as a pairs of self-equilibrated variable nodal forces. The “residual response” generated by the distortions superposed with the linear response equals to the modified response of the system. By analogy, distortions imposed on the model of electrical circuit can be interpreted as a virtual current sources associated with those elements, whose parameter modifications are modelled. The residual response can be found by solving equation (7) with the appropriate vector of distortions (time-dependent) on the right-hand side of the equation. However, in IVDM this is done in a simpler way by the superposition of linear combinations of distortions $\boldsymbol{\varepsilon}$ making use of the so-called *impulse influence matrix* **D**. The formula for the residual response of the system (voltages) is:

$$u_i^R(t) = \sum_{\tau=0}^{t} \sum_j D_{ij}(t-\tau)\varepsilon_j(\tau) \tag{8}$$

The impulse influence matrix is a three-dimensional matrix which stores the responses of the system on the impulse excitations. The member $D_{ij}(t_0)$ gives the voltage response in the i^{th} edge of the circuit in the instant $t = t_0$ on unit impulse distortion imposed on the j^{th} edge in the instant $t = 0$. Relation (8) is based on the

principle that the time-variable function (and so are distortions grouped in the vector **ε**) can be considered as a sequence of impulses equal to the values of function at the given instants. Influence matrix is calculated using Newmark algorithm (given column is a vector of response for the impulse excitation in the appropriate edge), so the time dimension is also discrete. The relation for current residual response can also be derived (similarly to the equation (8) with the difference that influence matrix needs to store current responses).
As it was mentioned, distortions model the modifications of the system. In the following analysis it is assumed that only the conductance of elements can change. The constitutive relations for the modified and modelled circuit are:

$$\mathbf{i}(t) = \mathbf{C^e u'}(t) + \mathbf{G^{e\prime} u}(t) + \mathbf{K^e} \int \mathbf{u}(t) \tag{9}$$

$$\mathbf{i}(t) = \mathbf{C^e u'}(t) + \mathbf{G^e u}(t) + \mathbf{K^e} \int \mathbf{u}(t) + \boldsymbol{\varepsilon}(t) \tag{10}$$

where $\mathbf{i}(t)$, $\mathbf{u}(t)$ are vectors of modified responses of the circuit, $\mathbf{G^{e\prime}}$ is a matrix of modified values of conductance and $\boldsymbol{\varepsilon}(t)$ is a vector of distortion. The interpretation is that distortions compensate the change of currents in elements caused by the change of conductance. Comparing (10) and (9), according to the claim that responses of modified and modelled circuits are the same, the following relation is obtained:

$$\varepsilon_i(t) = (\mu_i - 1) G^e_{ii} u_i(t) \tag{11}$$

where $\mu_i = G^{e\prime}_i \,/\, G^e_i$ is modification parameter. On the right-hand side of the relation (11) we have the vector of modified response $\mathbf{u}(t)$, which is the sum of linear and residual response, so it also depends on distortions:

$$\mathbf{u}(\boldsymbol{\varepsilon},t) = \mathbf{u^L}(t) + \mathbf{u^R}(\boldsymbol{\varepsilon},t) \tag{12}$$

The conclusions coming from the analysis of relations (11) and (12) are that distortions occur only in those elements of modelled system, where modifications occur in modified system, but their form also depends on other distortions imposed on the system. Inserting (12) into (11) the following system of equation can be obtained:

$$\sum_{\tau=0}^{t} \sum_{j} [\delta_{t\tau}\delta_{ij} - (\mu_i - 1) G^e_{ij} D_{ij}(t-\tau)] \varepsilon_j(\tau) = (\mu_i - 1) G^e_{ij} u^L_i(t) \tag{13}$$

It can be solved sequentially for every instant starting from t = 0, but it is numerically inefficient because of the fact that in every instant on the left-hand side there is another matrix which needs to be decomposed and inverted. Relation (13) can be transformed into another system of linear equations:

$$\mathbf{A^0}\, \boldsymbol{\varepsilon}(t) = \mathbf{b}(t) \tag{14}$$

$$A^0_{ij} = \delta_{ij} - (\mu_i - 1) G^e_{ij} D_{ij}(0) \tag{15}$$

$$b_i(0) = (\mu_i - 1)\sum_j G^e_{ij} u^L_j(0) \tag{16}$$

$$b_i(t > 0) = (\mu_i - 1)\sum_j G^e_{ij}[u^L_j(t) + u^R_j(t-1)] \tag{17}$$

System of equation (14) also has to be solved sequentially, but the matrix $\mathbf{A^0}$ is now constant for every instant t, only vector **b**(t) has to be updated. System of equations (14) allows calculating distortions that model the modification of the system, knowing beforehand the influence matrix and the linear response of the system on the determined excitation. Distortions imposed on the initial system will generate the same response as in the modified system.

THE IDENTIFICATION PROCEDURE (CONCEPT)

The purpose of the proposed procedure is identification of defects, interpreted as modifications of conductance in certain elements of the system. The procedure relies on the examination of changes in the response of the system subjected to the determined excitation. The choice of excitation function is a problem by itself and will not be discussed here, but generally the function should be chosen in such a way to obtain the response of the modified system significantly different from the linear response. The proposed procedure of defect identification is gradient based and reduces to the problem of optimisation the objective function *g* which base on the summation of differences between the measured response $u^M(t)$ of the modified system and the response $u(t,\mu)$ of the modelled system, with the modification parameter μ as the steering variable:

$$g(\mu) = \sum_\alpha \sum_t [u_\alpha(t,\mu) - u^M_\alpha(t)]^2 \tag{18}$$

Gradient of the objective function with respect to the steering variable:

$$\frac{\partial g(\mu)}{\partial \mu_s} = 2\sum_\alpha \sum_t [u_\alpha(t,\mu) - u^M_\alpha(t)]\frac{\partial u_\alpha(t,\mu)}{\partial \mu_s} \tag{19}$$

The main part of calculations is finding the gradient of the response function. We will make use of the chain rule:

$$\frac{\partial u_\alpha(t,\mu)}{\partial \mu_s} = \frac{\partial u_\alpha(t,\varepsilon)}{\partial \varepsilon_p(t,\mu)} \cdot \frac{\partial \varepsilon_p(t,\mu)}{\partial \mu_s} \tag{20}$$

The gradient of system response with respect to the selected distortion is easy to calculate making use of the equations (12) and (8):

$$\frac{\partial u_\alpha(t)}{\partial \varepsilon_p(t)} = \sum_{\tau=0}^{t} D_{\alpha p}(t-\tau)\varepsilon_p(\tau) \tag{21}$$

The formula for the gradient of distortion with respect to the modification parameter is derived as a result of differentiation of equation (14) with respect to the selected modification parameter.

$$A^0_{ij}\frac{\partial \varepsilon_i}{\partial \mu_s}(t) = b^s_i(t) \tag{22}$$

Matrix $\mathbf{A^0}$ remains unchanged (relation 15) and vectors on the right-hand side are expressed as follows:

$$b^s_i(0) = \sum_j G^e_{ij} u_j(0) \tag{23}$$

$$b^s_i(t>0) = \sum_j G^e_{ij} u_j(t) + (\mu_i - 1)\sum_{\tau=0}^{t-1}\sum_j G^e_{ij} D_{ij}(t-\tau)\frac{\partial \varepsilon_j(\tau)}{\partial \mu_s} \tag{24}$$

System of equation (24) needs to be solved sequentially for every time step and for every chosen parameter μ_s.

The algorithm of optimisation base on the steepest descent method. In every iteration the vector of steering variable is updated according to the following (general) formula:

$$\mu^{(i+1)} = \mu^{(i)} - \frac{\partial g(\mu)}{\partial \mu}\cdot\frac{\Delta}{N} \tag{25}$$

where N is a certain norm dependent on gradient and Δ is a step length (which also can be optimised in order to speed up the convergence). The general concept of algorithm is presented in fig. 2.

CONCLUSION

It has been demonstrated that basing on the system of analogy certain class of electrical circuits can be analysed utilizing the method from structural mechanics. The presented derivations give a theoretical basis for the future development and practical application. The presented identification procedure is up-to-now only a concept as work on programming it is still in progress.

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Initial data:

- FEM-based model of the circuit
- Reference excitation function
- Measured response of modified circuit

Initial computations:

- Linear response
- Impulse influence matrix

Iterative procedure:

- finding the gradient (relations 20-22) and the value of the response function for the given vector of steering variables
- finding the gradient of the objective function (relation 19)
- finding the successive vector of steering variables (relation 25)
- checking the constraints imposed on steering variables
- checking the stop criterion

Figure 2. General algorithm of defect identification

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